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LETTER TO THE EDITOR

The non-triviality of the one-dimensional problem of a 'true' self-avoiding walk

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Abstract. It is shown that the root-mean-square distance $\overline{R^2}^{1/2}$ for the self-avoiding walk problem is related to the number of steps N as $\overline{R^2} \sim N^{2\nu}$ with $\nu = \frac{2}{3}$ rather than the 'self-obvious' $\nu = 1$. This coincides with the mean-field theory result obtained by Pietronero which erroneously was not extended to the case $d = 1$.

It has been shown recently by Amit *et al* (1983) that the statistics of the self-avoiding walk differs from that of a polymer chain with excluded volume. It was shown that the upper critical dimension for this problem is $d_c = 2$ instead of $d_c = 4$ for the problem of a chain with excluded volume. The diagram technique rules and renormalisation group equations for this problem at $d \leq 2$ were obtained by Obukhov and Peliti (1983). Using a quite different Flory-like self-consistent approximation, Pietronero (1983) obtained a simple formula for the mean-square distance exponent:

$$\begin{aligned} \nu &= \tfrac{1}{2}, \qquad && d \geq 2 \\ \nu &= 2/(2+d), \qquad && d < 2. \end{aligned} \tag{1}$$

At dimension $d = 2 - \varepsilon$ slightly smaller than two this result disagrees with the result $\nu = \frac{1}{2} + \frac{1}{2}\varepsilon + O(\varepsilon^2)$ obtained by the ε-expansion method as is usually the case for the Flory method near the upper critical dimension. According to Pietronero (1983), (1) is inapplicable also at $d = 1$, since at $d = 1$ the problem seems to have a trivial solution with $\nu = 1$.

However, the Flory method usually gives a very good approximation for ν at low dimensions and the exact value of ν at marginal dimension. It will be shown here that at $d = 1$ the self-avoiding walk problem is non-trivial and is characterised by $\nu = \frac{2}{3}$ in agreement with (1).

The most general formulation of the self-avoiding problem can be made using five independent charges (Bulgadaev and Obukhov 1983), but in the one-dimensional case only one charge is relevant. The probability of each step from point i to the neighbouring points $i \pm 1$ can be written in the form

$$P_{i\pm1} = \exp(-gh_{i\pm1})/[\exp(-gn_{i+1}) + \exp(-gn_{i-1})]. \tag{2}$$

Here $n_{i\pm1}$ is the number of previous visits of points $i \pm 1$. If $g \gg 1$ the probability to turn back for a walker who starts from the origin in one direction is exponentially small. Thus, if the number of steps $N \ll e^g$, then $\nu = 1$. But, if $N \sim e^g$ then the turning points may well appear. If once the walker turns backward, the probability that he

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turns once more is again e^{-g}. But the whole walk cannot be considered as a random one with an effective step $N_0 \sim e^g$. It is because the turning points are the points where the density of previous visits n has a discontinuity of height two, and these points affect very strongly the further wandering.

Now we consider the case of small g. Let the wandering begin at point $x=0$ and $P_N(x)$ be the probability that after N steps the walker is at point x. Then for $P_{N+1}(x)$ we can write, expanding (2)

$$P_{N+1}(x) = P_N(x-1)(\tfrac{1}{2} - \nabla n_N(x-1)g) + P_N(x+1)(\tfrac{1}{2} + \nabla n_N(x+1)g). \tag{3}$$

Here $n_N(x)$ is the number of visits of point x after N steps, and ∇ is a lattice derivative, $\nabla n(x) = \frac{1}{2}[n(x+1) - n(x-1)]$. Expanding $P_{N+1}(x)$ and $n_N(x \pm 1)$ we obtain

$$\frac{\partial P_N(x)}{\partial N} = \frac{1}{2}\frac{\partial^2 P_N(x)}{\partial x^2} + 2gP_N(x)\frac{\partial^2 n_N(x)}{\partial x^2} + 2g\frac{\partial P_N}{\partial x}\frac{\partial n_N}{\partial x}. \tag{4}$$

For $n_N(x)$ we have an obvious normalisation condition

$$\int_{-\infty}^{+\infty} n_N(x)\,\mathrm{d}x = N.$$

The first term on the RHS of (4) is comparable with the others and should be retained only in the case of a random walk when $\bar{x}^2 \sim N$. This is possible when the number of previous visits is small, i.e. $ng \ll 1$ or $N \ll 1/g^2$. If the reverse inequality holds, $N \gg 1/g^2$, this term can be omitted and through the dimensionality arguments we obtain

$$1/N \sim (n/x^2)g, \qquad \text{where } n \sim N/x$$

or

$$x \sim g^{1/3}N^{2/3}, \qquad n \sim g^{-1/3}N^{1/3}. \tag{5}$$

Using (5) it can be shown that $\nabla n_N(x) \sim g^{1/3}N^{-1/3} \ll 1$, i.e. the expansion of the exponent in (2) in derivation (3) was made correctly.

Thus the corelation length exponent is $\nu = \frac{2}{3}$ in agreement with Pietronero's result at $d=1$.

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